Final report on “Stability of Intensively Evaporating Liquid Film with Co-current Gas Flow”

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I. INTRODUCTION

Convection occurring in a horizontal liquid layer has received extensive attention since Bénard observed hexagonal roll cells upon onset of convection in a layer of molten spermaceti with a free surface [1]. The phenomenon of convection in an evaporating liquid layer are of particular interest because of its importance in heat exchangers, distillation, drying technologies and cooling of microelectronic equipments.

During evaporation, an essential mechanism is that evaporation leads to intensive cooling of the liquid-vapor interface. When the temperature drop induced by evaporation across the liquid layer exceeds a critical value, convective instability occurs. This instability mechanism has been studied by many previous investigators.

In recent years, increasing performance demands in semiconductor technology, including shrinking feature size, increasing transistor density, and faster circuit speeds, have resulted in very high chip power dissipation and heat fluxes. It is also leading to greater non-uniformity of on-chip power dissipation, creating localized, sub-millimeter hot spots, often exceeding $1\mathtt{kW/cm}^2$ in heat flux, which can degrade the processor performance and reliability [2]. Similar developments are underway in microwave integrated circuits and power amplifier chips, with even higher localized heat fluxes and heat densities. The industrial and technological applications mentioned above involve thin liquid films on uniform or non-uniform heated substrates. To avoid the reduction of their performance by film breakdown it is of crucial importance to understand when and why instabilities arise that may result in rupture of the film. Understanding the physical mechanisms of instability and rupture behavior is also highly desirable for the requirement of seeking effective ways to suppress the rupture of heated films.

Shear-induced flows of liquid films are important for a number of technological innovations for ground and space applications. A particularly promising technological candidate to prevent the rupture of film due to temperature gradients at the interface, allowing to reach high heat fluxes and to minimize space and mass of cooling equipment, is a set-up where heat is transferred to subcooled thin liquid film driven by a forced gas on one side of a mini-channel. It is quite evident that the combined effects of evaporation, thermocapillarity, gas dynamics, and gravity as well as the formation of microscopic adsorbed film on the wall, are somewhat complicated issues and have not yet been studied systematically.

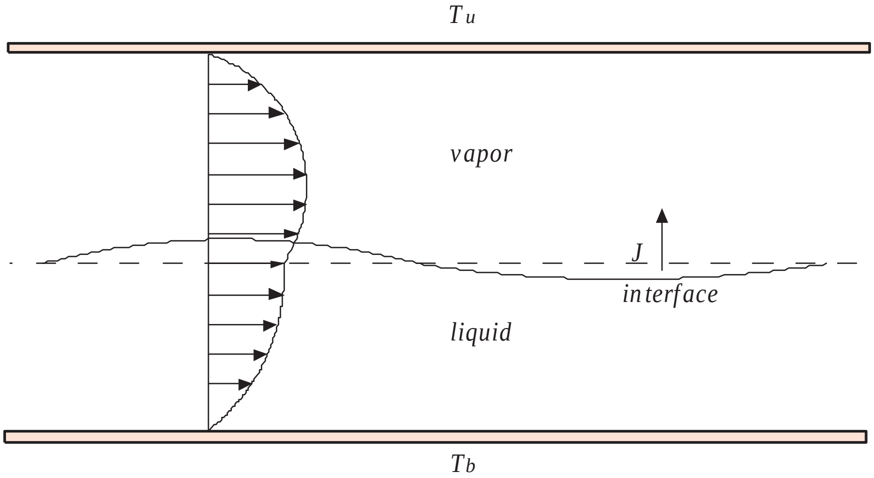

FIG. 1: A sketch of the physical model for problem 1.

From July 2011 to now, three problems have been investigated. The progresses are list below:

1.Instabilities in a horizontal liquid layer in co-current gas flow with an evaporating interface;

2.Instability of a locally heated falling film subjected to an interfacial shear;

3.Effect of mutual location of heaters on the stability of thin films flowing over locally heated surfaces.

II. PROGRESS OF THE RESEARCH ACTIVITIES

A. Problem 1

We consider a two-layer system consisting of a liquid layer of thickness d_l underlying its own vapor of thickness d_v as shown in Fig.(1). The system is infinite in the streamwise (x) and the spanwise (y) directions. The acceleration of gravity is opposite to the z direction. We assumed that the top wall and the bottom wall are rigid perfectly conducting boundaries. The gas flow is driven by constant pressure gradient. The vapor and liquid phases are separated by an immiscible and deformable interface where phase change can occur. The surface tension σ decreases linearly with the temperature T, i.e. $\sigma = \sigma_0 - \sigma_T(T - T_0)$, where σ_T is a constant coefficient and T_0 the temperature of reference state.

The goal is to investigate the influences of interfacial shear and evaporation on the stability of the system. We chose the system consisting of a water layer in contact with its own vapor at 100 $^{\circ}C$. The physical properties of the liquid and the vapor phase are the same with that in Ref.[3]. The ratios of physical properties are $\rho^* = 6.25 \times 10^{-4}$, $\nu^* = 71.72$, $\chi^* = 3.68 \times 10^{-2}$ and $\kappa^* = 0.118$. The Prandtl number $Pr = 1.78$.

The physics of the coupling of evaporation, interfacial shear, Rayleigh effect and Marangoni effect is complicated. When investigate the problem, we will look at different aspects of the problem separately whenever possible.

1. *Rayleigh-Bénard instability*

We begin with the influence of evaporation and interfacial shear on the Rayleigh convection in the system. The depth ratio is an important parameter to influence the Rayleigh instability of the system. The marginal curves of the critical Rayleigh number versus the wavenumber are shown in Fig.2.

In order know more about the characteristics of the Rayleigh convection of the liquid mode and the vapor mode, we plot the flow fields of these two modes on the onset of convection in Fig.3.

2. *Marangoni-Bénard instability*

In this subsection, we will study the influence of evaporating and interfacial shear on the Marangoni instability. Fig.4 shows the effect of non-equilibrium degree at the interface on the Marangoni instability of the system. In order to know the influence of evaporation effect and the interfacial shear on the coupling mode between the vapor and liquid layers, we plot the flow patterns on the onset of shortwave Marangoni instability in Fig.5.

3. *Conclusions*

We have studied the instability of a horizontal liquid layer in co-current gas flow with an evaporating interface. In this system, the interfacial shear is induced by the co-current gas flow. We focus on the effects of evaporation and the interfacial shear on the Rayleigh and Marangoni instabilities.

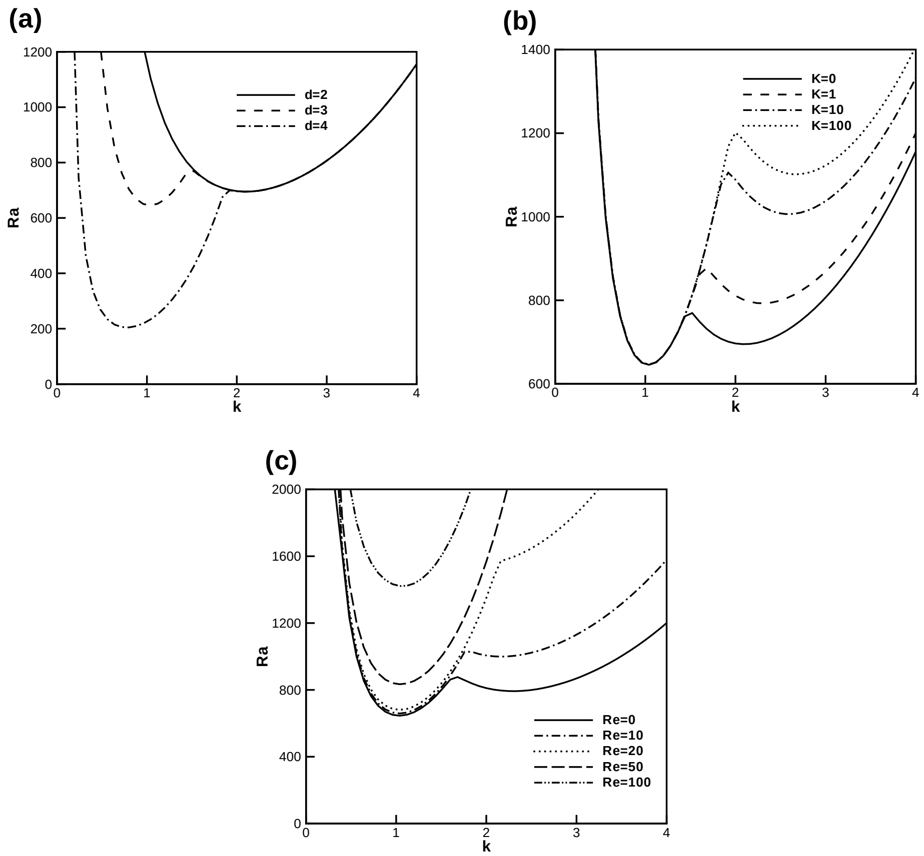

FIG. 2: (a) Effect of the depth ratio d on the marginal curves of the critical Rayleigh number versus the wavenumber at $K = 0$; (b) Effect of K on the marginal curves of the critical Rayleigh number versus the wavenumber at $d = 3$; (c) Effect of the Reynolds number Re on the marginal curves of the critical Rayleigh number versus the wavenumber at $d = 3$, $K = 1$.

For the Rayleigh problem, convection can initiate in the vapor layer or in the liquid layer, depending on the depth ratio d. The results show that the change of d only influences the vapor mode, and the change of the parameter K, which represents the non-equilibrium degree at the interface, only influences the critical Rayleigh number of the liquid mode.

Both the evaporation and the interfacial shear play important roles in the Marangoni instability. The parameter K has different influences on the longwave mode and the shortwave mode of Marangoni instability. With the increase of K, the short wave mode becomes more stable and the longwave mode becomes more unstable. The influence of the interfacial shear also has different effects on the shortwave and the longwave Marangoni instability. With the increase of Re the shortwave Marangoni mode becomes more stable and the longwave

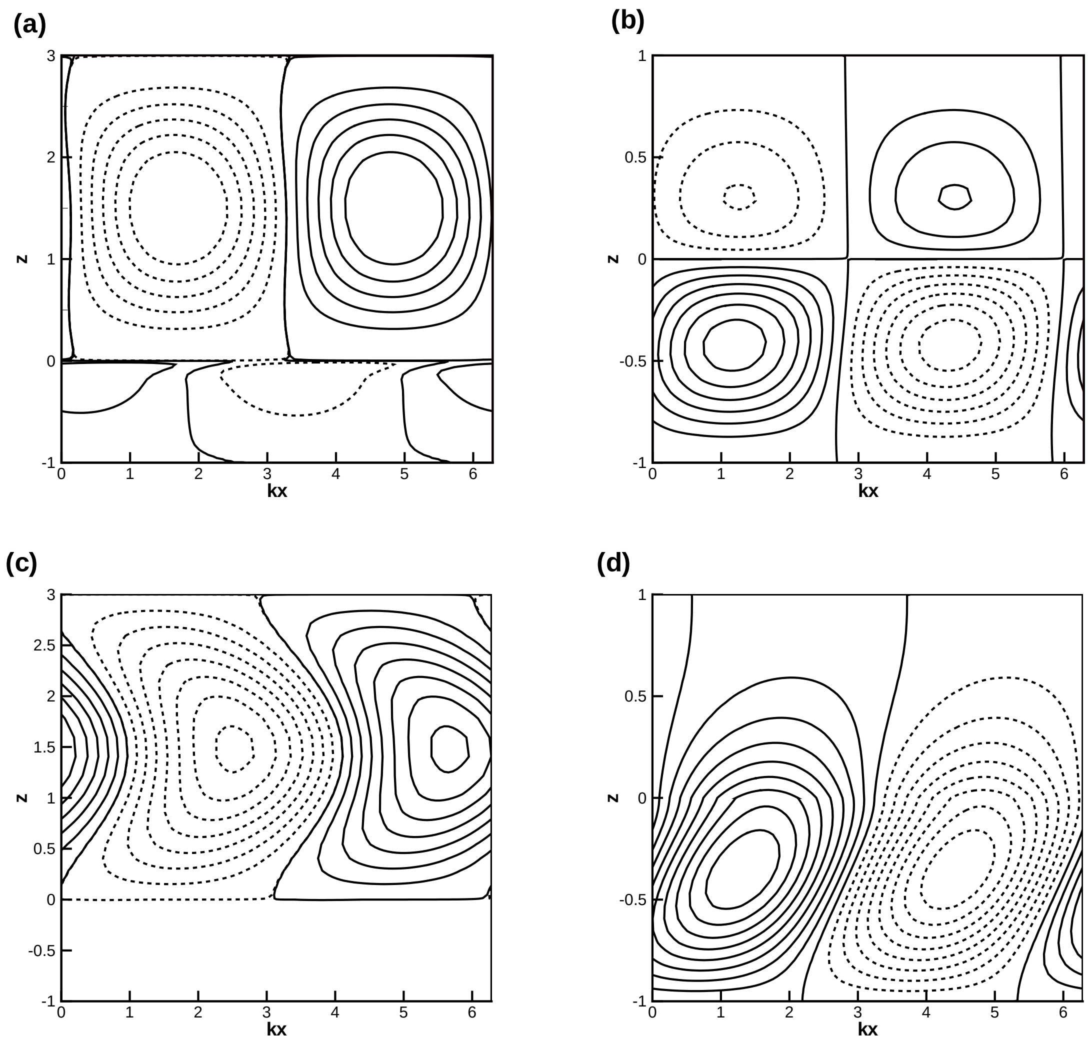

FIG. 3: The flow fields of streamfunction of disturbance initiating in the vapor layer (a) and the liquid layer (b); (c), (d) the isothermal lines for temperature disturbance. The parameters for (a), and (c) are $Ra = 1421.8$, $k = 1.04$, $d = 3$, $K = 1$ and $Re = 100$; The parameters for (b), (d) are $Ra = 984.1$, $k = 2.18$, $d = 1$, $K = 1$ and $Re = 10$.

mode becomes more unstable.

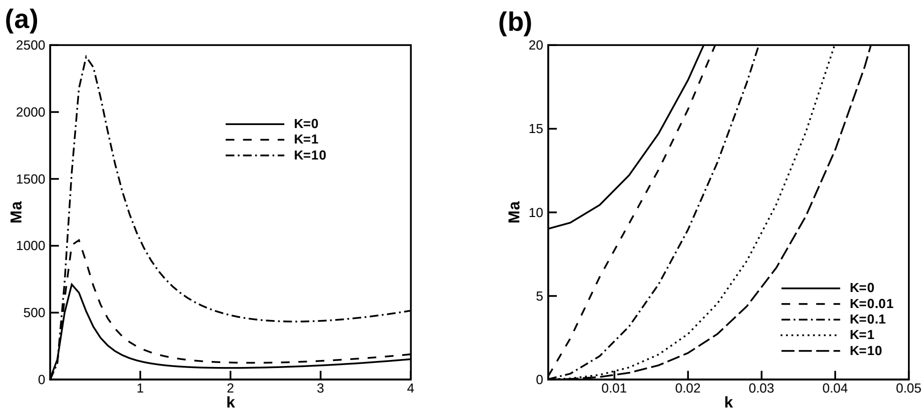

FIG. 4: (a) Effect of K on the marginal curves of the critical Marangoni number versus the wavenumber at $d = 1$, $Re = 0$;(b) Effect of K on the marginal curves of the critical Marangoni number versus the wavenumber in the longwave range at $d = 1$, $Re = 0$.

B. Problem 2

1. *description of the physical model*

We investigate the influence of mutual location of heaters on the dynamics of a thin film falling down a locally heated plate. The longwave approximation is used to reduce the Navier-Stokes equations and free-surface boundary conditions to a nonlinear partial differential equation for the evolution of the local height of the free surface. We will studied two typical cases, i.e. the heaters with rectangular shapes and with infinite spanwise lengths. We solved the 2-D and 3-D steady states by Newtonian iteration method for these two cases, respectively. Moreover, we studied the linear stability and nonmodal stability of locally heated films for these two typical cases.

Consider a thin liquid film falling down an inclined substrate with inclination angle θ with respect to the horizontal direction. The coordinate system is constructed with x in the streamwise direction, y the spanwise direction and z normal to the substrate. A heater is embedded in the substrate and produces the temperature field $T_0(x, y)$ at the plate surface, and the thickness of film far away from the heater is uniform. The thickness of the film is $h(x, y)$.

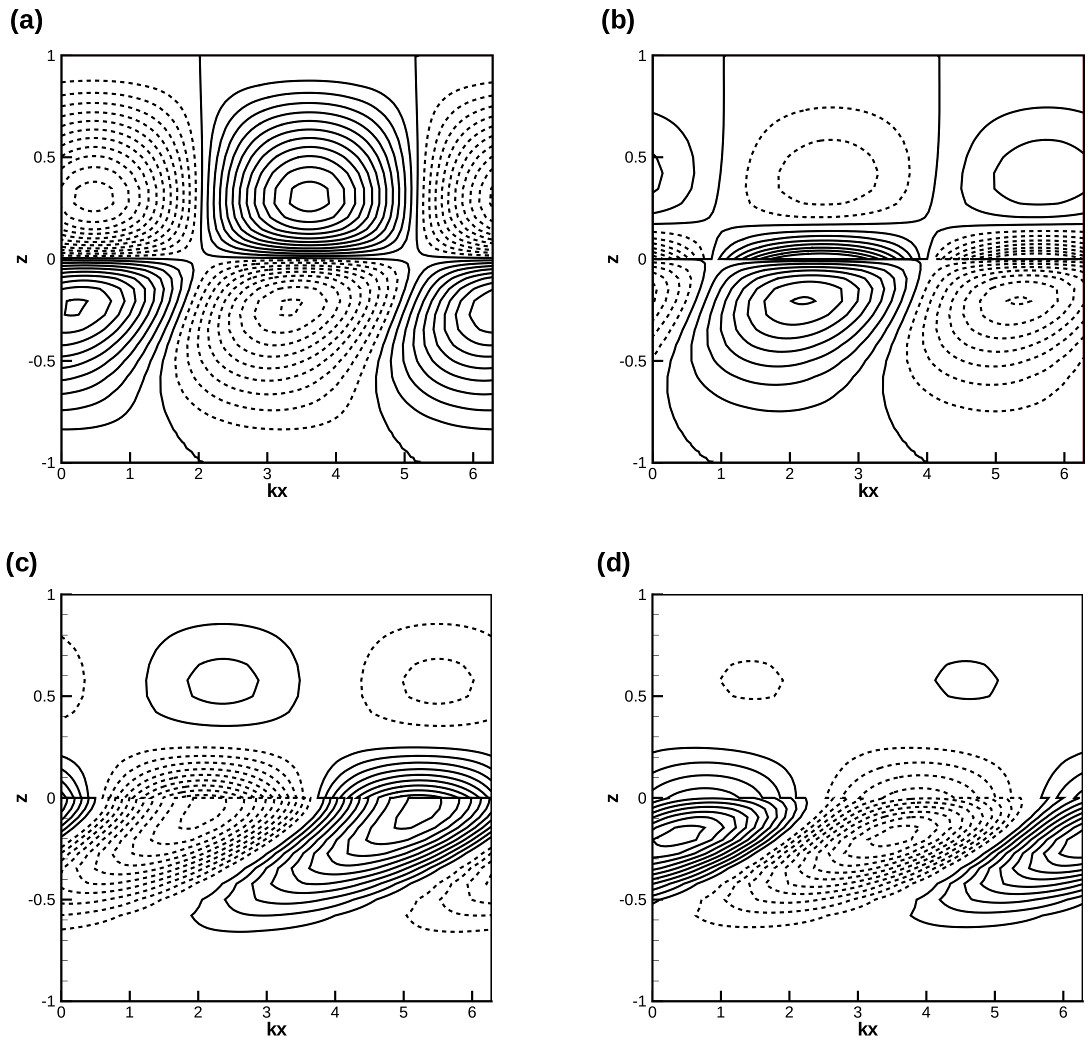

FIG. 5: (a), (b) The flow fields of streamfunction of disturbance; (c), (d) the isothermal lines for temperature disturbance. The parameters for (a),(b) are $Ma = 410.5$, $\alpha = 2.44$, $d = 1$, $K = 0$ and $Re = 100$; the parameters for (b),(d) are $Ma = 1159.9$, $\alpha = 3.56$, $d = 1$, $K = 10$ and $Re = 100$.

The evolution equation of the thickness of the film is expressed as

$$\frac{\partial h}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{h^3}{3Bo} \boldsymbol{\nabla} \nabla^2 h - \frac{h^2}{2} Ma \boldsymbol{\nabla} T_i \right) + \frac{1}{3} \frac{\partial h^3}{\partial x} = 0. \tag{1}$$

In the present paper, we will study the influence of the shape and the mutual location on the dynamics of the thin film. For a heater with a finite streamwise length and an infinite spanwise length, the temperature can be prescribed as

$$T_0(x) = f(x) = 0.5[\tanh(x - L_x/2) - \tanh(x + L_x/2)]. \tag{2}$$

For a heater with a rectangular shape, the temperature can be prescribed as

$$T_0(x,y) = f(x,y) = 0.25[\tanh(x-L_x/2)-\tanh(x+L_x/2)][\tanh(y-L_y/2)-\tanh(y+L_y/2)]. \tag{3}$$

For a couple of heaters arranged in the streamwise direction with a distance of d, the temperature can be given as

$$T_0(x,y) = f(x-d,y) + f(x,y). \tag{4}$$

2. results and discussions

In Fig.6, base profiles are plotted for a thin film heated by a single heater and a couple of heaters with infinite spanwise lengths. As shown in Fig.6(a) and (b), at the upstream edge of the heaters, the Marangoni stress induced by temperature gradient opposes the flow of the film, resulting in the thickening of film to maintain a constant flow rate and forming pronounced thermocapillary ridges. In the case of only one heater, a bump structure has been formed in these two figures. In the case of a couple of heaters, at $d = 1$ only one bump has been formed. As the distances increases to $d = 2$, two bumps has been formed at the upstream edge of the two heaters. The amplitude of the downstream bump is smaller than that of the upstream one. As the distance increases to $d = 4$, it is found that the two bumps almost have the same height. These bumps are unstable to infinitesimal disturbance. We will study the influence of the mutual location on the stability of the film in the next section.

In order to know the effect of mutual location of heaters on the structure of steady profile, we plot in Fig.7 the contour of thickness of the film heated by a couple of heaters. In Fig.7(a) for heaters with $l_x = l_y = 2$, two lateral waves are formed from the first heater and extend to a long distance in the downstream direction. The film thickens in the upstream direction near the first heater and in the lateral waves. The position of the minimum thickness of film occurs in the downstream direction near the second heater. In Fig.4(b) and 4(c) for heaters with wider spanwise lengths, the film thickens in the upstream directions near the two heaters. Two lateral waves are formed from the second heater. In Fig.7(a),(b) and (c), the position of the minimum film thickness always occurs in the downstream direction near the second heater.

In order to know the influence of mutual location and the shape of heater on the rupture

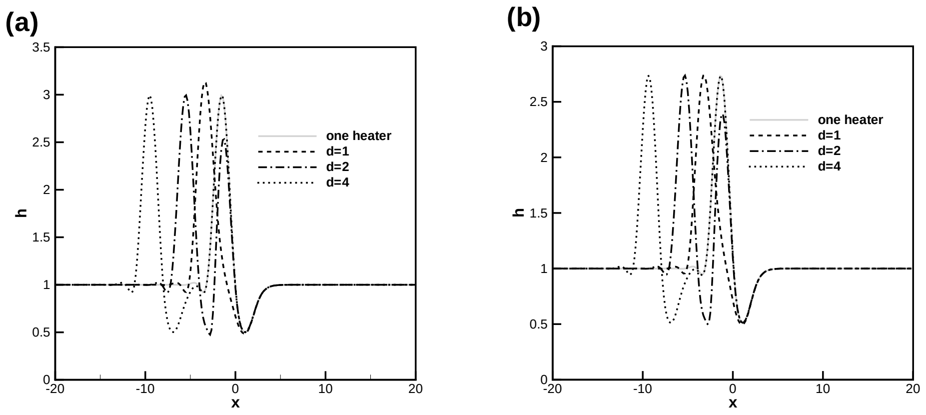

FIG. 6: The film profiles for the steady solutions. (a) $Bi = 0$ (b) $Bi = 0.1$. The other parameters are $Ma = 5.0$ and $Bo = 10$.

of the film, we plot in Fig.8 the minimum thickness of the film at various Marangoni numbers for various ratios $r = l_y/l_x$. In Fig.8(a) for films heated by one heater, the minimum thickness decreases with the increase of the Marangoni number. As the Marangoni number approaches to a certain value (Ma_c), the minimum thickness decreases dramatically to zero. For $r = 1$, the Maragoni number at which film rupture occurs is slightly greater than 1. In this figure, Ma_c increases with the increase of r. As r increase to 8, film rupture occurs at $Ma_c \approx 3$. This result shows that increasing the spanwise length of the heater can inhibit the rupture of the film. In Fig.8(b), the curves of minimum thickness of films heated by a couple of heaters are plotted for various r. Comparing the curves in Fig.8(b) and Fig.8(a), we find that film rupture occurs at a lower Marangoni number for a film heated by two heaters than by one heater.

3. Conclusions

The problem is studied in the framework of longwave theory. We studied the influence of mutual location of heaters on the steady state of the film. As the film is heated by heaters with infinite spanwise lengths, 2D steady states exist. As the film is heated by rectangular heaters, the steady state display a typical 3D structure. In the case of streamwise arrangement of the heaters of rectangular shape film rupture is most likely on the second heater. A linear stability analysis is performed with respected to both the 2D and 3D basic state. The results show that the mutual location of heater plays an important role in the

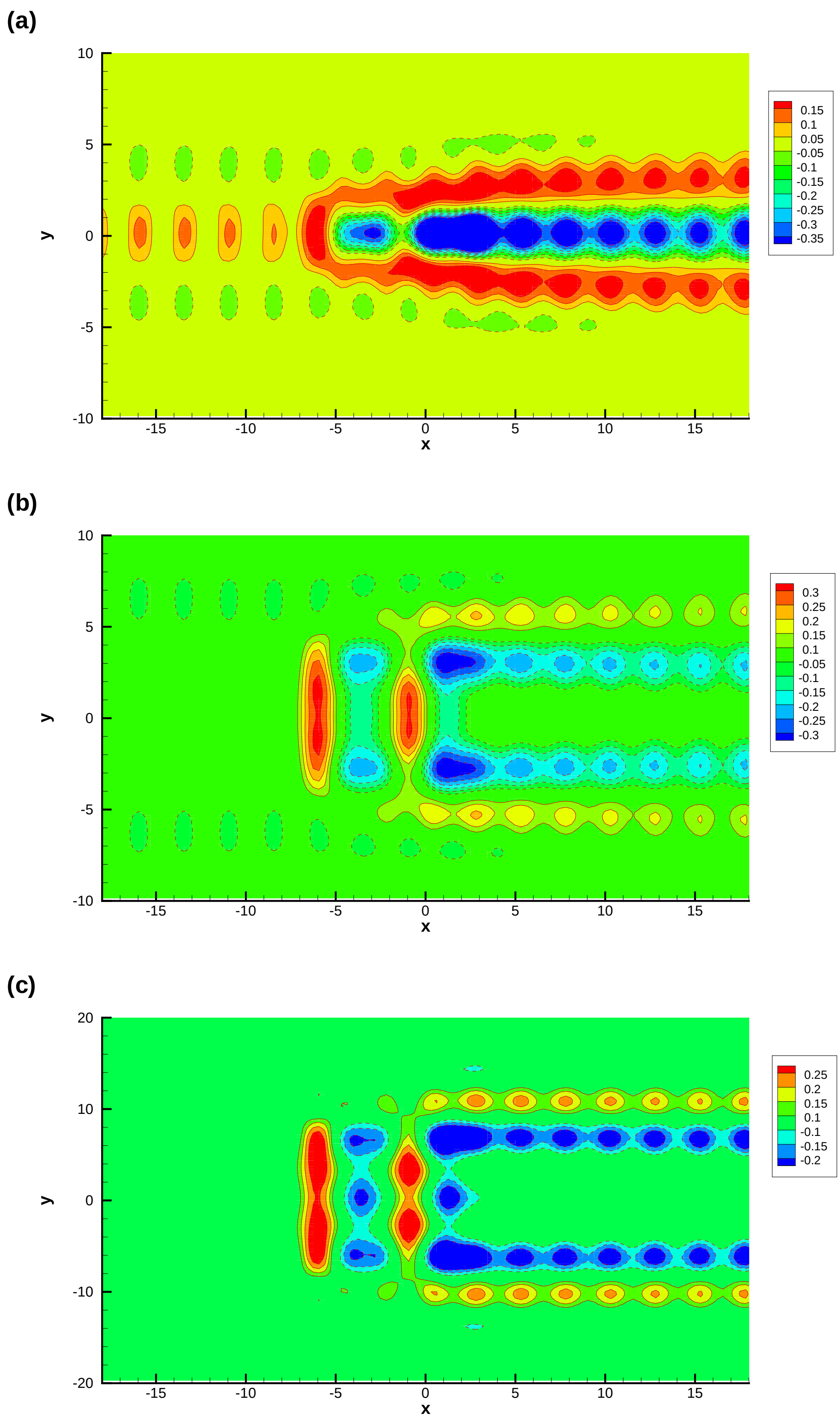

FIG. 7: The 3-D structure of the surface of film heated by a pair of heaters. The size of the heater is (a) $l_x = 2$, $l_y = 2$, (b) $l_x = 2$, $l_y = 8$, (c) $l_x = 2$, $l_y = 16$. The other parameters are $Ma = 1$, $Bi = 0$ and $Bo = 10$.

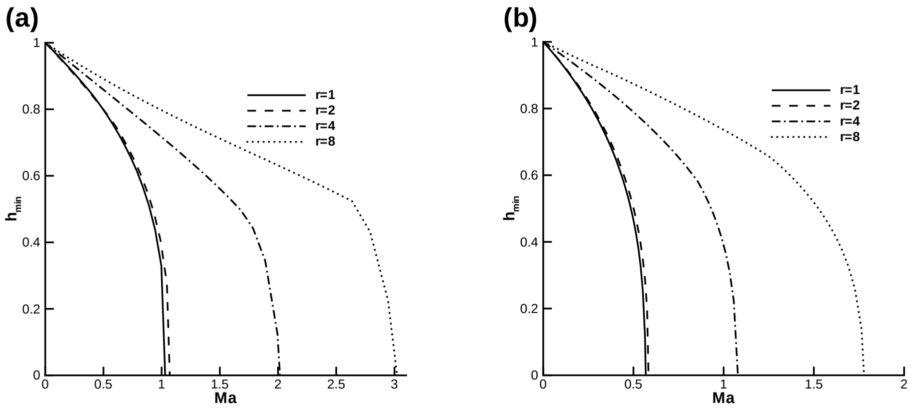

FIG. 8: The curves of the minimum thickness versus the Marangoni number. (a) one heater, (b) a couple of heaters. The other parameters are $Bi = 0$ and $Bo = 10$.

stability and transient behavior of the film for 2D steady state. However, the 3D steady state of a film heated by rectangular heaters is linear stable. This means that the 3D steady state is a coherent structure in films heated by rectangular heaters.

C. Problem 3

We consider the motion of a liquid film falling down a locally heated planar substrate. The flow is driven by gravity and a unidirectional 'wind' shear $\boldsymbol{\tau}$ is applied to the free surface. The problem is studied in the framework of longwave theory. Marangoni effect due to the local temperature gradients at the free surface induces a horizontal bump in the vicinity of the upper edge of the heater and results in an instability in the form of a rivulet structure periodic in the transverse direction. We focus on the effect of shear $\boldsymbol{\tau}$ on the two-dimensional steady-state solutions of longwave equations. Further we analyse the linear stability of this bump with respect to disturbances in the spanwise direction. Our computations show that the shear in streamwise directions will decrease the hight of the bump. We also studied the influence of $\boldsymbol{\tau}$ on the dispersion relations between growth rate of the fastest growth mode and the wavenumber. It is shown that the increase of $\boldsymbol{\tau}$ significantly damps the growth rate of the most unstable mode. Three-dimensional simulations have been performed to investigate the influence of shear on development of the rivulet structure beyond the instability threshold.

1. results and discussions

We obtain the evolution equation of the thickness of the film,

$$\frac{\partial h}{\partial t} + \boldsymbol{\nabla} \cdot \left(\frac{h^3}{3Bo} \boldsymbol{\nabla} \nabla^2 h - \frac{h^2}{2} Ma \boldsymbol{\nabla} T_i \right) + \frac{1}{2} \boldsymbol{\nabla} \cdot (\boldsymbol{\tau} h^2) + \frac{1}{3} \frac{\partial h^3}{\partial x} = 0. \tag{5}$$

We aim to seek the 2-D steady state of $\frac{\partial}{\partial t} = 0$ as the base state for 3-D instability analysis. We assume that for the steady state the flow has no variation in the spanwise direction, i.e. $\frac{\partial}{\partial y} = 0$, it is obvious that the shape of h for 2-D steady state is independent of the spanwise component of the interfacial shear. In the present study, two typical cases of temperature profiles at the substrate surface are used to investigated the instability of the film, i.e. a simi-infinite heater and a finite-length heater. The temperature profile of a semi-infinite heater is

$$T_0 = 0.5[1 + \tanh(\frac{2l}{\pi L} \sin(\frac{\pi L}{l} x))], \tag{6}$$

with $L \ll l$ and the wavelength $\lambda = \frac{l}{L}$. A periodic temperature gradient at the plate surface is prescribed as

$$\frac{\partial}{\partial x} T_0 = \mid \cos(\pi x L/l) \mid \mathrm{sech}^2(\frac{2l}{\pi L} \sin(\frac{\pi L}{l} x)). \tag{7}$$

The influence of interfacial shear on the base profile of the interface is presented in Fig.9. It is shown that with the increase of the interfacial shear the hight of the bump decreases. In Fig.10, the influence of the interfacial shear on the dispersion relation are presented. It is shown that the interfacial shear is stabilizing.

III. PERSPECTIVE

We have studied three problems related to stability of evaporating liquid film with co-current gas flow. In general, we use classical Newton's cooling law in one-sided model or the Hertz-Knudsen relation in two-sided model to describe the evaporating interface. In our further research, we will use more general boundary conditions to describe the non-equilibrium relation at the interface.

Acknowledgments

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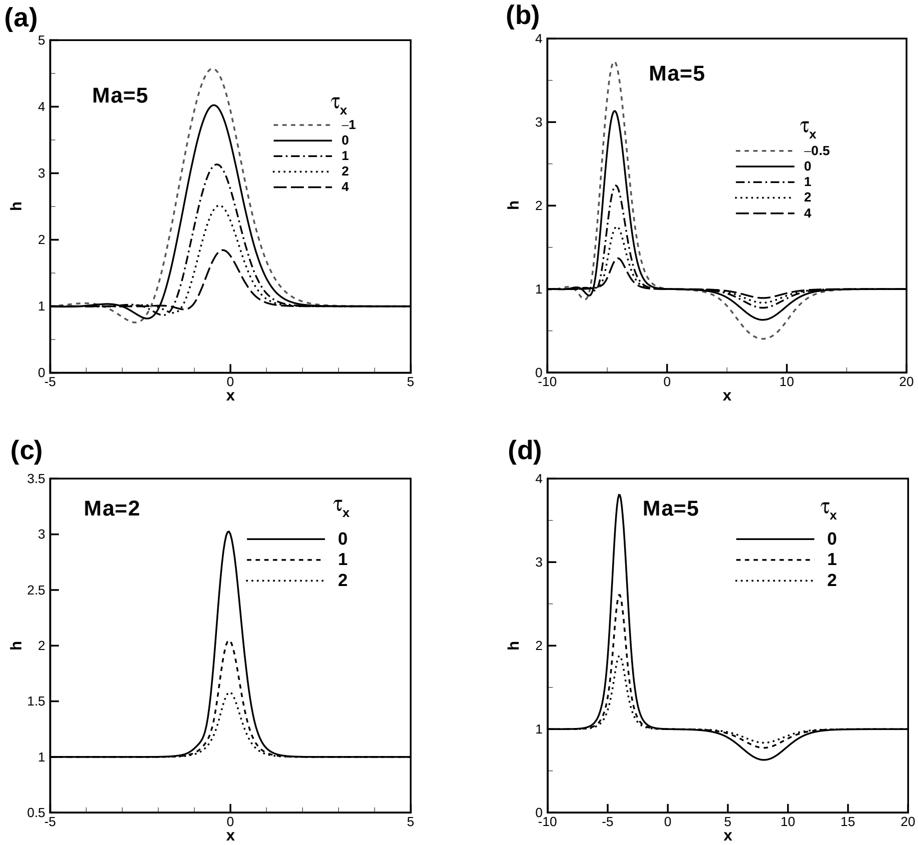

FIG. 9: Effect of interfacial shear on the film profiles for the steady solutions at $Bi = 0$. (a),(c) for semi-infinite heater with $l = 20L$ and the wavelength $\lambda = l/L$, (b),(d) for finite length heater with the wavelength $\lambda = 40$. $Bo = 10$ for (a) and (b), and $Bo = 1000$ for (c) and (d).

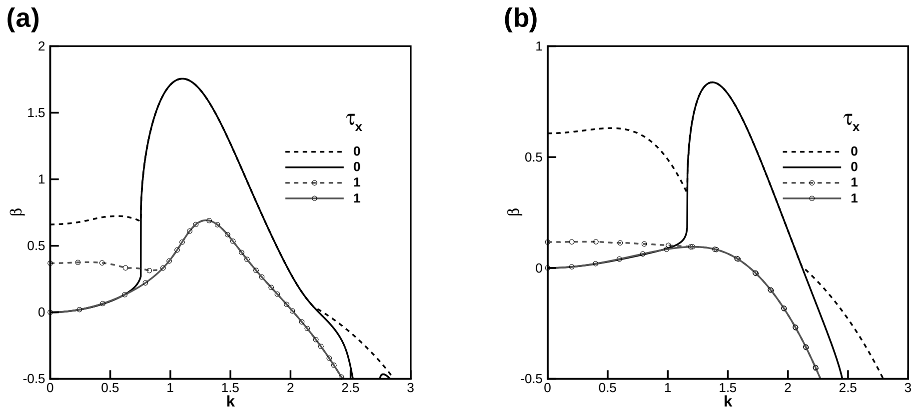

FIG. 10: Effect of interfacial shear on dispersion relations of the growth rate β versus the wavenumber k (a) for semi-infinite length heater with the wavelength $\lambda = 20$, $Ma = 7.9$, $Bi = 0.1$, $Bo = 10$. (b) for finite length heater with the wavelength $\lambda = 40$, $Ma = 7.9$, $Bi = 0.1$, $Bo = 10$.

IV. A LIST OF PUBLICATIONS

R. Liu and O. A. Kabov, "Instabilities in a horizontal liquid layer in co-current gas flow with an evaporating interface", Physical Review E, 2012, (accepted).

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